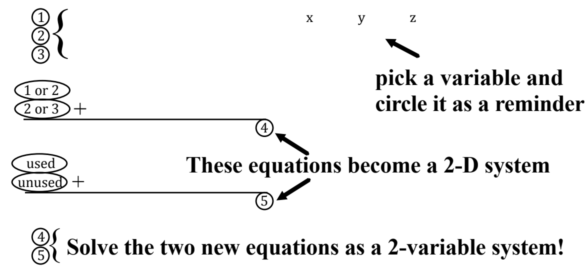
Name: ______________________________ Per: _____

Solving 3-D Systems

To solve 3-variable systems, you have to work in stages.

1. Pick a variable to eliminate.
2. Choose two 3-variable equations to eliminate the variable from, creating a 2-variable equation.
3. Pick another set of two 3-variable equations (one you've used, and the one you left out) and eliminate that variable again, creating a second 2-variable equation.
4. Now, you've got a 2-variable system. Substitute or eliminate to solve it.
5. Plug those two variables back into one of the original equations to determine the third variable.



EXAMPLE:

① ② ③ $\begin{cases} 2x + 4y - 3z = 11 \\ x + y + 3z = 18 \\ -x + 4y - 2z = -10 \end{cases}$ x (y) z

(1 or 2) $2x + 4y - 3z = 11$ → $2x + 4y - 3z = 11$
(2 or 3) $+ (-x + 4y - 2z = -10) (-1)$ → $+ x - 4y + 2z = 10$
④ $3x \quad - z = 21$

I multiplied the bottom equation by -1 so I can add down to cancel the y's

(used) $2x + 4y - 3z = 11$ → $2x + 4y \quad - 3z \quad = 11$
(unused) $+ (x + y \quad + 3z = 18)(-4)$ → $+ -4x - 4y \quad - 12z = -72$
⑤ $-2x \quad - 15z = -61$

I multiplied the bottom equation by -4 so I can add down to cancel the y's

④ ⑤ $\begin{cases} (3x - z = 21)(-15) \\ -2x - 15z = -61 \end{cases}$ → $-45x + 15z = -315$ → $+ -2x - 15z = -61$

$-47x \quad = -376$

$x = 8$

Now, I choose to eliminate the z's by Multiplying the top equation by -15

Plug in x = 8...

$3x - z = 21$
$3(8) - z = 21$
$24 - z = 21$
$-z = -3$
$z = 3$

Plug in x = 8, and z - 3...

$2x + 4y - 3z = 11$
$2(8) + 4y - 3(3) = 11$
$16 + 4y - 9 = 11$
$7 + 4y = 11$
$4y = 4$
$y = 1$

The solution to the system is:

(8, 1, 3)

EXAMPLE:

① ② ③ $\begin{cases} 5x + y + 2z = 3 \\ 10x + 2y + 6z = -6 \\ 20x + 4y + 8z = 8 \end{cases}$ x y (z)

(1 or 2) $(5x + y + 2z = 3) (-3)$ → $-15x - 3y - 6z = -9$
(2 or 3) $+ 10x + 2y + 6z = -6$ → $+ 10x + 2y + 6z = -6$
④ $-5x - y \quad = -15$

(used) $(5x + y + 2z = 3) (-4)$ → $-20x - 4y - 8z = -12$
(unused) $+ 20x + 4y + 8z = 8$ → $+ 20x + 4y + 8z = 8$
⑤ $0 = -4$

④ ⑤ $\begin{cases} -5x - y = -15 \\ \end{cases}$ → →

STOP!!!!

There is NO SOLUTION!!
The system is INCONSISTENT!!

Solve the systems shown below.

1.

① ② ③	$\begin{cases} -6x - 2y + 4z = -8 \\ 3x + y + 2z = 12 \\ -x + 3y - 2z = 0 \end{cases}$	x	y	z

(1 or 2) →

(2 or 3) + → +

④

(used) →

(unused) + → +

⑤

④ { →

⑤ { →

2.

① ② ③	$\begin{cases} 3x - 2y - z = 10 \\ 3x + 3y + 3z = 12 \\ 2x + y - 10z = 30 \end{cases}$	x	y	z

(1 or 2) →

(2 or 3) + → +

④

(used) →

(unused) + → +

⑤

④ { →

⑤ { →

Solve the systems shown below.

3.

		x	y	z
① ② ③	$\begin{cases} -5x - 3y - 4z = 10 \\ 10x + 6y + 8z = -22 \\ 5x + 3y + 4z = 10 \end{cases}$			

1 or 2 →

2 or 3 + → +

④

used →

unused + → +

⑤

④ ⑤ { → →

4.

		x	y	z
① ② ③	$\begin{cases} 2x + 5y - 2z = 9 \\ 2x + y + 3z = 3 \\ 4x - 15y + 5z = 25 \end{cases}$			

1 or 2 →

2 or 3 + → +

④

used →

unused + → +

⑤

④ ⑤ { → →

Check each system. Is (-3, 5, 2) a solution?

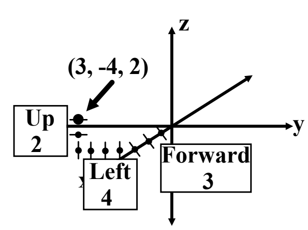
5. $\begin{cases} 2x - 4y + z = 12 \\ 3x + 5y - 2z = 12 \\ 8x + y + z = -17 \end{cases}$	6. $\begin{cases} -9x + 5y - 10z = 32 \\ 7x - 5y + 8z = 12 \\ x + y + z = 4 \end{cases}$	7. $\begin{cases} -4x + 4y - 4z = 0 \\ 4x - 4y + 4z = -24 \\ 4x + 4y + 4z = 40 \end{cases}$
8. $\begin{cases} 7x + y - z = -18 \\ x + 6y - z = 25 \\ 2x + 3y + z = 11 \end{cases}$	9. $\begin{cases} 6x - 6y + 2z = -44 \\ 2x + 3z = 0 \\ 3y - 4z = 7 \end{cases}$	10. $\begin{cases} x + 2y + 6z = 19 \\ x = -3 \\ 4x + y = -7 \end{cases}$

Now, let's graph some 3-D points. Pay attention, there is a new axis (z), and the other two have changed direction. **X** is now traveling **backward (-)** and **forward (+)**. **Y** is now traveling **left (-)** and **right (+)**. **Z** is traveling **down (-)** and **up (+)**. Because we're working in 3-D, you can't just count out the steps and plot the point anymore. It's too hard to figure out where that point is in 3-D space. So, you have to leave a trail of breadcrumbs like Hansel and Gretel. There are no witches at the end of this story, though—I promise. Each unit you move gets a dot.

For example, **(3, -4, 2)** gets

3 dots moving **forward** along the x-axis,
4 dots moving **left** (because y is negative) parallel to the y-axis,
and **2** dots moving **up** parallel to the z-axis.

The final dot is your point: (3, -4, 2)



You try.

11. (-5, 2, 1)	12. (3, 4, 2)	13. (6, 7, -3)