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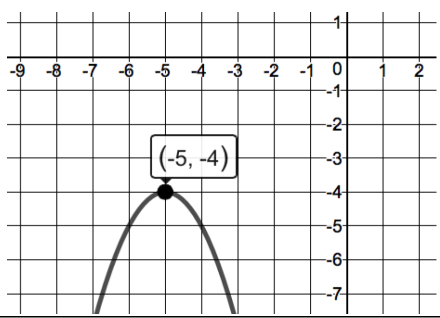
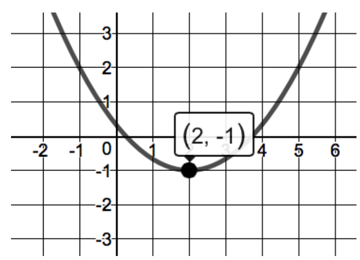
Determining the Vertex, Transformations and the Graph

In Vertex Form: $f(x) = a(x-h)^2 + k$
The vertex of a parabola is found at (h, k). To find it, look to the equation: h is the opposite of what's inside behind x, but k is the same of what's outside at the end.

Steps to Identify the Vertex	EXAMPLE 1: **Determine the vertex.** $\boldsymbol{f(x) = -2(x+5)^2 - 4}$	EXAMPLE 2: **Determine the vertex.** $\boldsymbol{g(x) = \frac{1}{3}(x-2)^2 - 1}$
Look at the equation. $f(x) = a(x-h)^2 + k$ Circle the number where h is and the number where k is.	$f(x) = a\left(x\boxed{-h}\right)^2\boxed{+k}$ $f(x) = -2\left(x\boxed{+5}\right)^2\boxed{-4}$	$f(x) = a\left(x\boxed{-h}\right)^2\boxed{+k}$ $f(x) = \frac{1}{3}\left(x\boxed{-2}\right)^2\boxed{-1}$
The vertex is (h, k), is made from the opposite sign of what h says it is, and the same as what k says it is.	$f(x) = -2\left(x\boxed{+5}\right)^2\boxed{-4}$ $Vertex: \left(\boxed{-5}, \boxed{-4}\right)$	$f(x) = \frac{1}{3}\left(x\boxed{-2}\right)^2\boxed{-1}$ $Vertex: \left(\boxed{+2}, \boxed{-1}\right)$

Steps to Graph the Parabola	EXAMPLE 1 (cont'd): **Graph.** $\boldsymbol{f(x) = -2(x+5)^2 - 4}$	EXAMPLE 2 (cont'd): **Graph.** $\boldsymbol{g(x) = \frac{1}{3}(x-2)^2 - 1}$
Step 1: Identify the vertex (we did that already…see above). Step 2: Plot the vertex on the graph. Step 3: Check if the parabola opens up (+) or down (-). Step 4: Draw the curve in that direction.	Step 1: The vertex is $(-5, -4)$ Step 2: Plot it. Steps 3 & 4: It's $-$, so draw it down. 	Step 1: The vertex is $(2, -1)$ Step 2: Plot it. Steps 3 & 4: It's $+$, so draw it up. 

Identifying the Transformations	EXAMPLE 1 (cont'd): **Describe the transformations.** $\boldsymbol{f(x) = -2(x+5)^2 - 4}$	EXAMPLE 2 (cont'd): **Describe the transformations.** $\boldsymbol{g(x) = \frac{1}{3}(x-2)^2 - 1}$
*A negative in front means it reflects over the x-axis. *A number that is not 1 in the front is a vertical dilation by that number. *The vertex is the honest version of translations left/right (x-value) & translations up/down (y-value).	*There is a negative in the front of the equation, so the graph reflects over the x-axis. *There is a 2 (it doesn't matter that it's negative—that's the reflection) in front, so the graph is vertically dilated by 2. *The vertex is $(-5, -4)$, so the graph was translated left 5 ($x\ is\ -5$) and translated down 4 ($y\ is\ -4$).	*There **is no negative** in the front of the equation, so the graph doesn't reflect. *There is a $\frac{1}{3}$ in front, so the graph is vertically dilated by $\frac{1}{3}$. *The vertex is $(2, -1)$, so the graph was translated right 2 ($x\ is\ +2$) and translated down 1 ($y\ is\ -1$).

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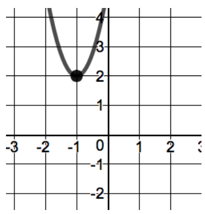
EXAMPLE $f(x) = 3(x+1)^2 + 2$
a. Identify the vertex
b. Graph the parabola
c. Describe the transformations

a. $f(x) = 3\left(x\boxed{+1}\right)^2\boxed{+2}$
The vertex is: $\left(opp\boxed{+1}, same\boxed{+2}\right)$
The vertex is: $(-1, 2)$

b. *Plot the vertex at* $(-1, 2)$*, and, because the equation is +, draw the graph curving up from that point.*

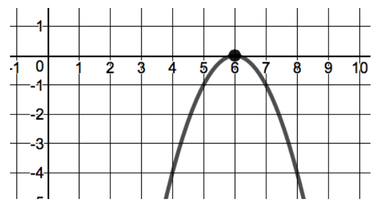


c. $f(x) = 3(x+1)^2 + 2$
*It's positive, so it has no reflection (no negative=no reflection).
*The number out front (3) is not 1, so it is vertically dilated by 3.
*The vertex is $(-1, 2)$, so it is translated left 1 ($x = -1$) and translated up 2 ($y = +2$).

EXAMPLE $f(x) = -(x-6)^2$
a. Identify the vertex
b. Graph the parabola
c. Describe the transformations

a. $f(x) = -\left(x\boxed{-6}\right)^2\boxed{}$
The vertex is: $\left(opp\boxed{+6}, same\boxed{}\right)$
The vertex is: $(6, 0)$

b. *Plot the vertex at* $(6, 0)$*, and, because the equation is* $-$*, draw the graph curving down from that point.*

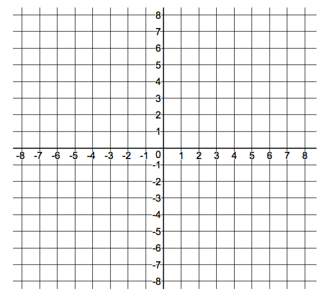


c. $f(x) = -(x-6)^2$
*It's negative, so there is a reflection over the x-axis.
*The number out front is invisible (which means it's 1), so the graph is not vertically dilated.
*The vertex is $(6, 0)$, so it is translated right 6 ($x = 6$) and not translated up or down ($y = 0$).

1. $\boldsymbol{f(x) = 2(x+5)^2 - 3}$
a. Identify the vertex
b. Graph the parabola
c. Describe the transformations

a.

b.

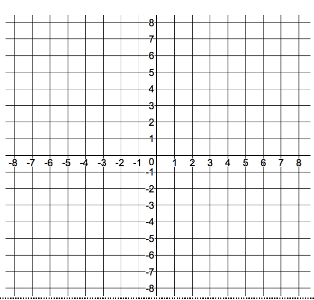


c.

2. $\boldsymbol{f(x) = -\frac{1}{4}(x-1)^2 - 2}$
a. Identify the vertex
b. Graph the parabola
c. Describe the transformations

a.

b.

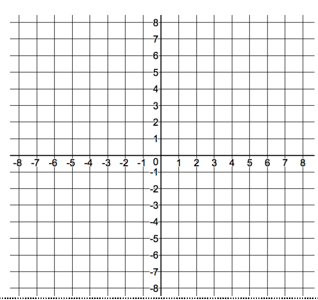


c.

3. $\boldsymbol{f(x) = (x-4)^2 + 3}$
a. Identify the vertex
b. Graph the parabola
c. Describe the transformations

a.

b.

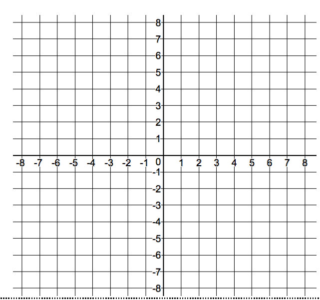


c.

4. $\boldsymbol{f(x) = -(x)^2 + 1}$
a. Identify the vertex
b. Graph the parabola
c. Describe the transformations

a.

b.



c.

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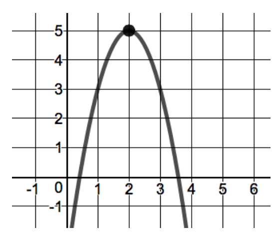
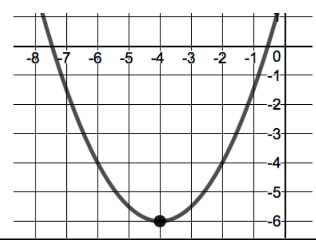
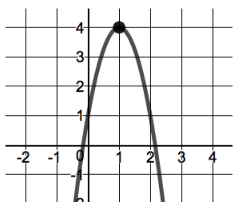
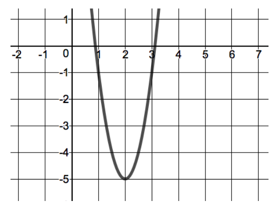
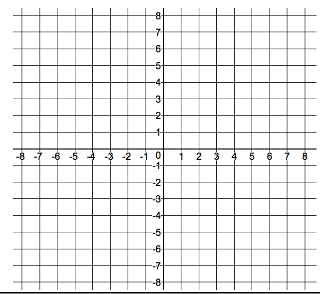
In Standard Form: $f(x) = ax^2 + bx + c$

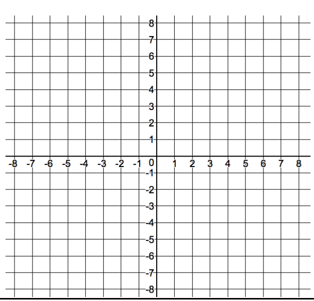
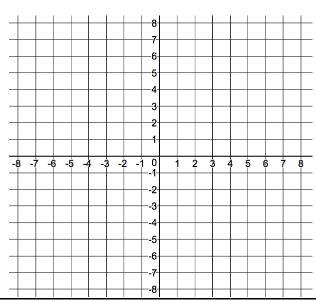
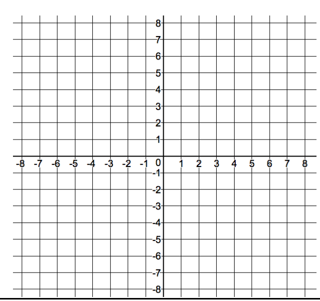
The vertex of the parabola is found at $\left(-\frac{b}{2a}, y \text{ when you plug in } \left(-\frac{b}{2a}\right)\right)$.

Steps to Identify the Vertex	EXAMPLE 1: **Determine the vertex.** $\boldsymbol{f(x) = -2x^2 + 8x - 3}$	EXAMPLE 2: **Determine the vertex.** $\boldsymbol{g(x) = \frac{1}{2}x^2 + 4x + 2}$
Use the equation, $f(x) = ax^2 + bx + c$, and the formula for axis of symmetry $x = -\frac{b}{2a}$ to find the x-value	$f(x) = -2x^2 + 8x - 3$ $a = -2, b = 8, c = -3$ $x = -\frac{b}{2a}$ $x = -\frac{(8)}{2(-2)} = -\frac{8}{-4} = \frac{8}{4} = 2$ So far, the vertex is at $(2, ?)$	$g(x) = \frac{1}{2}x^2 + 4x + 2$ $a = \frac{1}{2}, b = 4, c = +2$ $x = -\frac{b}{2a}$ $x = -\frac{(4)}{2\left(\frac{1}{2}\right)} = -\frac{4}{1} = -4$ So far, the vertex is at $(-4, ?)$
Plug in the x-value you found and solve the equation to find the y-value.	$x = 2$ & $f(x) = -2x^2 + 8x - 3$ So… $f(2) = -2(2)^2 + 8(2) - 3$ $f(2) = -2(4) + 16 - 3$ $f(2) = -8 + 13$ $f(2) = 5$ So the vertex is: $(2, 5)$	$x = -4$ & $g(x) = \frac{1}{2}x^2 + 4x + 2$ So… $g(-4) = \frac{1}{2}(-4)^2 + 4(-4) + 2$ $g(-4) = \frac{1}{2}(16) - 16 + 2$ $g(-4) = 8 - 14$ $g(-4) = -6$ So the vertex is: $(-4, -6)$

Steps to Graph the Parabola	EXAMPLE 1 (cont'd): **Graph.** $\boldsymbol{f(x) = -2x^2 + 8x - 3}$	EXAMPLE 2 (cont'd): **Graph.** $\boldsymbol{g(x) = \frac{1}{2}x^2 + 4x + 2}$
Step 1: Identify the vertex (we did that already…see above). Step 2: Plot the vertex on the graph. Step 3: Check if the parabola opens up (+) or down (-). Step 4: Draw the curve in that direction.	Step 1: The vertex is $(2, 5)$ Step 2: Plot it. Steps 3 & 4: It's $-$, so draw it down. 	Step 1: The vertex is $(-4, -6)$ Step 2: Plot it. Steps 3 & 4: It's $+$, so draw it up. 

Identifying the Transformations	EXAMPLE 1 (cont'd): **Describe the transformations.** $\boldsymbol{f(x) = -2x^2 + 8x - 3}$	EXAMPLE 2 (cont'd): **Describe the transformations.** $\boldsymbol{g(x) = \frac{1}{2}x^2 + 4x + 2}$
*A negative in front means it reflects over the x-axis. *A number that is not 1 in the front is a vertical dilation by that number. *The vertex is the honest version of translations left/right (x-value) & translations up/down (y-value).	*There is a negative in the front of the equation, so the graph reflects over the x-axis. *There is a 2 (it doesn't matter that it's negative—that's the reflection) in front, so the graph is vertically dilated by 2. *The vertex is $(2, 5)$, so the graph was translated right 2 (x is $+2$) and translated up 5 (y is $+5$).	*There **is no negative** in the front of the equation, so the graph doesn't reflect. *There is a $\frac{1}{2}$ in front, so the graph is vertically dilated by $\frac{1}{2}$. *The vertex is $(-4, -6)$, so the graph was translated left 4 (x is -4) and translated down 6 (y is -6).

EXAMPLE $f(x) = -3x^2 + 6x + 1$ a. Identify the vertex b. Graph the parabola c. Describe the transformations	**EXAMPLE** $f(x) = 4x^2 - 16x + 11$ a. Identify the vertex b. Graph the parabola c. Describe the transformations	**5. $f(x) = -x^2 + 8x - 15$** **a. Identify the vertex** **b. Graph the parabola** **c. Describe the transformations**
a. $x = -\frac{b}{2a} = -\frac{(6)}{2(-3)} = -\frac{6}{-6} = +\frac{6}{+6}$ $x = 1$ Plug it in for *y:* $f(1) = -3(1)^2 + 6(1) + 1$ $f(1) = -3(1) + 6 + 1$ $f(1) = -3 + 7$ $f(1) = 4$ The vertex is at $(1, 4)$.	a. $x = -\frac{b}{2a} = -\frac{(-16)}{2(4)} = -\frac{-16}{8} = +\frac{+16}{8}$ $x = 2$ Plug it in for *y:* $f(2) = 4(2)^2 - 16(2) + 11$ $f(2) = 4(4) - 32 + 11$ $f(2) = 16 - 21$ $f(2) = -5$ The vertex is at $(2, -5)$.	a.
b. *Plot the vertex at* $(1, 4)$*, and, because the equation is* $-$*, draw the graph curving down from that point.* 	b. *Plot the vertex at* $(2, -5)$*, and, because the equation is +, draw the graph curving up from that point.* 	b. 
c. $f(x) = -3x^2 + 6x + 1$ * It's negative, so there is a reflection over the *x*-axis. *The number out front (3) is not 1, so it is vertically dilated by 3. *The vertex is $(1, 4)$, so it is translated right 1 $(x = +1)$ and translated up 4 $(y = +4)$.	c. $f(x) = 4x^2 - 16x + 11$ * It's positive, so it has no reflection (no negative=no reflection). *The number out front (4) is not 1, so it is vertically dilated by 4. *The vertex is $(2, -5)$, so it is translated right 2 $(x = +2)$ and translated down 5 $(y = -5)$.	c.

6. $f(x) = 5x^2 + 10x + 7$ **a. Identify the vertex** **b. Graph the parabola** **c. Describe the transformations**	**7. $f(x) = x^2 + 8x + 20$** **a. Identify the vertex** **b. Graph the parabola** **c. Describe the transformations**	**8. $f(x) = -3x^2 + 18x - 27$** **a. Identify the vertex** **b. Graph the parabola** **c. Describe the transformations**
a.	a.	a.
b. 	b. 	b. 
c.	c.	c.

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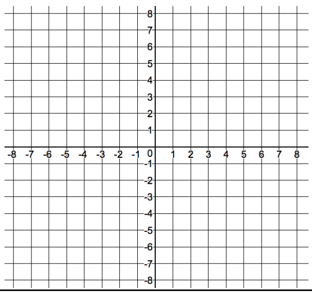
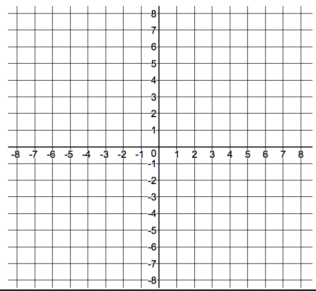
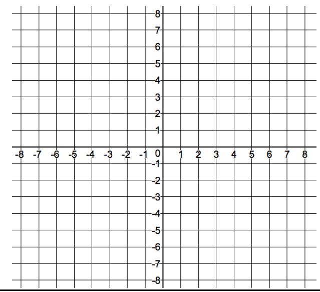
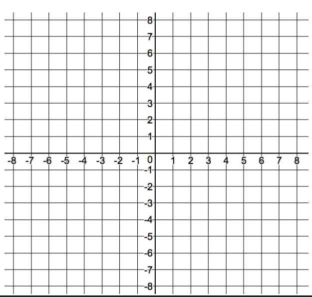
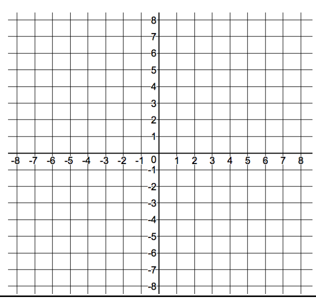
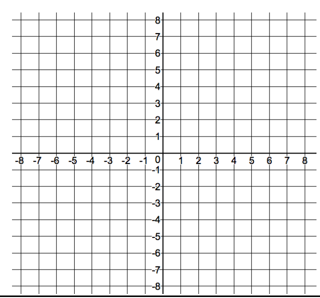
Mixed Practice

9. $f(x) = -3(x-2)^2 + 2$ **a. Identify the vertex** **b. Graph the parabola** **c. Describe the transformations**	**10.** $f(x) = 2x^2 + 8x + 3$ **a. Identify the vertex** **b. Graph the parabola** **c. Describe the transformations**	**11.** $f(x) = 5(x+1)^2 + 3$ **a. Identify the vertex** **b. Graph the parabola** **c. Describe the transformations**
a.	a.	a.
b. 	b. 	b. 
c.	c.	c.

12. $f(x) = -x^2 + 4x - 1$ **a. Identify the vertex** **b. Graph the parabola** **c. Describe the transformations**	**13.** $f(x) = \frac{1}{3}(x-4)^2$ **a. Identify the vertex** **b. Graph the parabola** **c. Describe the transformations**	**14.** $f(x) = -4x^2 - 8x + 2$ **a. Identify the vertex** **b. Graph the parabola** **c. Describe the transformations**
a.	a.	a.
b. 	b. 	b. 
c.	c.	c.

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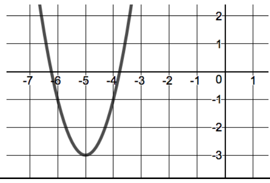
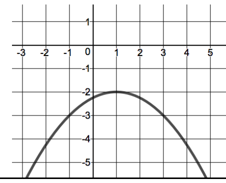
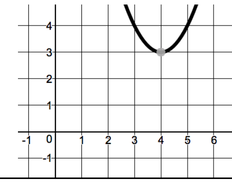
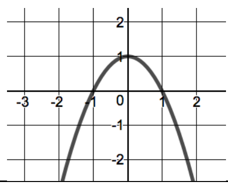
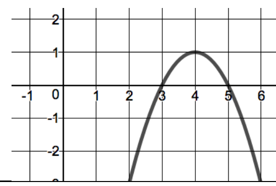
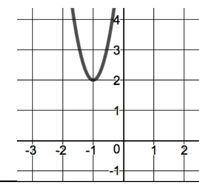
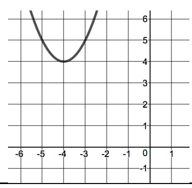
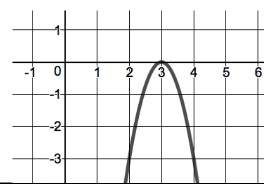
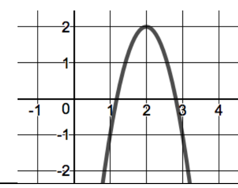
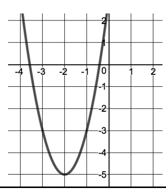
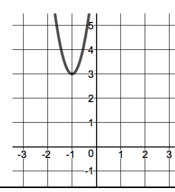
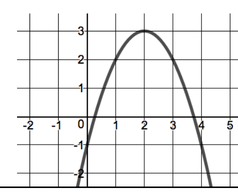
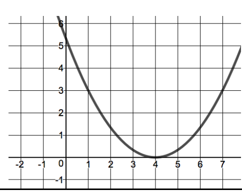
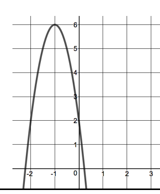
Make sure you check your answers against the **correct answers below**.

1a. $(-5,-3)$	2a. $(1,-2)$	3a. $(4,3)$
1b. 	2b. 	3b. 
1c. Vertically dilated by 2, translated left 5, and translated down 3.	2c. Reflected over the *x*-axis, vertically dilated by ¼, translated right 1, and translated down 2	3c. Translated right 4, and translated up 3

4a. $(0,1)$	5a. $(4,1)$	6a. $(-1,2)$
4b. 	5b. 	6b. 
4c. Reflected over the *x*-axis, and translated up 1.	5c. Reflected over the *x*-axis, translated right 4, and translated up 1.	6c. Vertically dilated by 5, translated left 1, and translated up 2.

7a. $(-4,4)$	8a. $(3,0)$	9a. $(2,2)$
7b. 	8b. 	9b. 
7c. Translated left 4, and translated up 4.	8c. Reflected over the *x*-axis, vertically dilated by 3, and translated right 3.	9c. Vertically dilated by 3, translated right 2, and translated up 2.

10a. $(-2,-5)$	11a. $(-1,3)$	12a. $(2,3)$
10b. 	11b. 	12b. 
10c. Vertically dilated by 2, translated left 2, and translated down 5.	11c. Vertically dilated by 5, translated left 1, and translated up 3.	12c. Reflected over the *x*-axis, translated right 2, and translated up 3.

13a. $(4,0)$	14a. $(-1,6)$
13b. 	14b. 
13c. Vertically dilated by 1/3, and translated right 4	14c. Reflected over the *x*-axis, vertically dilated by 4, translated left 1, and translated up 6.