Operations on Polynomials Practice

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|  | Addition and Subtraction Rules | Multiplication Rules |
|  | **Same** signs **Group Together****Different** Signs **Cancel Each Other** | **Same** signs **Make a Positive****Different** Signs **Make a Negative** |
| When the Signs **are the Same** | Two Positives Combine into a Positive:**+Number + Number = +Number** | Two Same Sign Numbers Multiply to a Positive:**(+Number)(+Number) = +Number**and **(–Number)(–Number) = +Number** |
| Two Negatives Combine into a Negative:**–Number – Number = –Number** |
| When the Signs **are Different** | A Positive and a Negative Combine to become the Bigger Number’s Sign:**–BigNumber + SmallNumber = –Number**or**–SmallNumber + BigNumber = +Number**or**+BigNumber – SmallNumber = +Number**or**+SmallNumber – BigNumber = –Number** | Two Different Sign Numbers Multiply to a Negative:**(+Number)(–Number) = –Number**and **(–Number)(+Number) = –Number** |

Combining with Variables:

Variables can only combine with variables that have the same exponent. In the table below, the only parts that can combine in each polynomial are the parts that are boxed, because they have the same exponents.

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| $$-7x^{2}+5$$$$-7x^{2}+5$$ | $$-4x^{3}+2x$$$$-4x^{3}+2x$$ | $$6x^{2}$$$$6x^{2}$$ | $$+5x^{2}$$$$+5x^{2}$$ |
| *Note: None of the exponents change, & only terms with* $x^{same exponent}$ *can combine.* |

Adding and Subtracting Polynomials

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| **EXAMPLE**$$\left(-5x^{3}+4x-5\right)-(-x^{2}+7x+6)$$**FIRST:** Change subtraction to addition and make inside opposite.$$\left(-5x^{3}+4x-5\right)\left(x^{2}7x6\right)$$**NEXT:** Bring together the like terms$$\left(-5x^{3}+4x-5\right)+\left(x^{2}-7x-6\right)$$$$-5x^{3}+x^{2}+\left(+4x-7x\right)+(-5-6)$$**FINALLY:** Simplify.$$-5x^{3}+x^{2}-3x-11$$ | **EXAMPLE**$$\left(4x^{2}-7x-3\right)+(7x^{2}-3x+2)$$*Don’t change it—it’s already adding!* **FIRST:** Bring together the like terms.$$\left(4x^{2}-7x-3\right)+\left(7x^{2}-3x+2\right)$$$$\left(4x^{2}+7x^{2}\right)+\left(-7x-3x\right)+(-3+2)$$**FINALLY:** Simplify.$$11x^{2}-10x-1$$ | 1. $\left(-x^{2}+3x-1\right)+(4x+5)$
 |
| 1. $\left(2x^{3}+8x^{2}-2\right)-(7x^{2}-3x+6)$
 | 1. $\left(-5x^{2}+9x+4\right)+(-5x+5)$
 | 1. $\left(x^{3}+7\right)-(-2x^{3}+5x+8)$
 |
| 1. $\left(5x^{2}-3x-6\right)+(9x-8)$
 | 1. $\left(6x-8\right)-(-x-8)$
 | 1. $\left(8x^{2}-4x+2\right)+(4x^{2}-7)$
 |

Multiplying with Variables:

When variables of the same base (*x* and *x, y* and *y*, etc.) multiply together, their exponents combine, because exponents tell us how many times to multiply by something.

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| $$(-x+2)(x-7)$$

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| --- | --- | --- |
|  | $$x$$ | $$-7$$ |
| $$-x$$ | $$-x^{2}$$ | $$+7x$$ |
| $$+2$$ | $$+2x$$ | $$-14$$ |

$$=-x^{2}+7x+2x-14$$$$=-x^{2}+9x-14$$ | $$(x^{2}+3x)(4x^{2}-5x)$$

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| --- | --- | --- |
|  | $$x^{2}$$ | $$+3x$$ |
| $$4x^{2}$$ | $$4x^{4}$$ | $$12x^{3}$$ |
| $$-5x$$ | $$-5x^{3}$$ | $$-15x^{2}$$ |

$$=4x^{4}+12x^{3}-5x^{3}-15x^{2}$$$$=4x^{4}+7x^{3}-15x^{2}$$ | $$(x+5)(-x^{2}+8)$$

|  |  |  |
| --- | --- | --- |
|  | $$x$$ | $$+5$$ |
| $$-x^{2}$$ | $$-x^{3}$$ | $$-5x^{2}$$ |
| $$+8$$ | $$+8x$$ | $$+40$$ |

$$=-x^{3}-5x^{2}+8x+40$$ | $$(-x-1)(-2x-7)$$

|  |  |  |
| --- | --- | --- |
|  | $$-2x$$ | $$-7$$ |
| $$-x$$ | $$+2x^{2}$$ | $$+7x$$ |
| $$-1$$ | $$+2x$$ | $$+7$$ |

$$=2x^{2}+7x+2x+7$$$$=2x^{2}+9x+7$$ |
| *Note: The product (multiplication answer) only had negatives when we multiplied different signs (+ & -) together.* |

Multiplying Polynomials

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| **EXAMPLE**$$(-5x-1)(6x+8)$$

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**FIRST:** Write what you’re multiplying on the table – it doesn’t matter which factor goes on the top or the left.

|  |  |  |
| --- | --- | --- |
|  | $$-5x$$ | $$-1$$ |
| $$6x$$ |  |  |
| $$+8$$ |  |  |

*Don’t forget the signs (+ & -)!!!*

|  |  |  |
| --- | --- | --- |
|  | $$-5x$$ | $$-1$$ |
| $$6x$$ | $$-30x^{2}$$ | $$-6x$$ |
| $$+8$$ | $$-40x$$ | $$-8$$ |

**NEXT:** Find the product in each box (multiply).$\left(-5x\right)\left(6x\right)=-30x^{2}$, $\left(-1\right)\left(6x\right)=-6x$,$\left(-5x\right)\left(8\right)=-40x$, $\& \left(-1\right)\left(8\right)=-8$**FINALLY:** Combine the products.$$=-30x^{2}-40x-6x-8$$$$=-30x^{2}-46x-8$$ | **EXAMPLE**$(6x^{2}+4)(7x-6)$

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**FIRST:** Write what you’re multiplying on the table – it doesn’t matter which factor goes on the top or the left.

|  |  |  |
| --- | --- | --- |
|  | $$7x$$ | $$-6$$ |
| $$6x^{2}$$ |  |  |
| $$+4$$ |  |  |

*Don’t forget the signs (+ & -)!!!*

|  |  |  |
| --- | --- | --- |
|  | $$7x$$ | $$-6$$ |
| $$6x^{2}$$ | $$42x^{3}$$ | $$-36x^{2}$$ |
| $$+4$$ | $$28x$$ | $$-48$$ |

**NEXT:** Find the product in each box (multiply).$\left(7x\right)\left(6x^{2}\right)=42x^{3}$, $\left(-6\right)\left(6x^{2}\right)=-36x^{2}$,$\left(7x\right)\left(+4\right)=28x$, $\& \left(-6\right)\left(+4\right)=-48$**FINALLY:** Combine the products.$$=42x^{3}-36x^{2}+28x-48$$ | 1. $(-4x+2)(-3x-5)$

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| 1. $(7x-6)(-3x+5)$

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 | 1. $(-7x^{2}+5x)(-4x-4)$

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 | 1. $(-x+6)(x+3)$

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| 1. $(x-7)(9x^{2}-6)$

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 | 1. $(2x+6)(6x+9)$

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 | 1. $(-5x-2)(-8x-3)$

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| *How to* ***Factor a Polynomial and Find the Roots*** | *Problem:* |
| *Thinking Step: Is the problem negative or have a number you can divide out?* | $15x^{2},12x, $*&* $3$ *can all be divided by 3.*$,$$ , \& $ | ***Write*** $f\left(x\right)=15x^{2}-12x-3$***in factored form, and determine the roots.*** |
| ***First Step:*** *If you found something in the thinking step…**factor it out!* | $$f\left(x\right)=15x^{2}-12x-3$$$$f\left(x\right)=3(5x^{2}-4x-1)$$ |
| *Thinking Step:**What are A, B, & C in the part you want to factor* $(Ax^{2}+Bx+C)$*?* | $$I want to factor: (5x^{2}-4x-1)$$$$A=5, B=-4, C=-1$$ |
| *Thinking Step:**What 2 numbers will multiply to equal AC & combine to equal B?* | $$AC=\left(5\right)\left(-1\right)=-5, \& B=-4$$$$ *multiply to* $AC=5$ *and combine to* $B=-4$ | $$f\left(x\right)=3\left(5x^{2}-4x-1\right)$$$$f\left(x\right)=3[5x^{2}-1]$$ |
| ***Second Step:*** *Split the middle x term into the two numbers from the last Thinking Step.* |
| *Thinking Step:* *What do the first two terms both divide by? What do the last two terms both divide by?* | $$5x^{2}+1x-5x-1$$$5x^{2}+1x$ *both divide by x*$$ *&* $$$-5x-1$ *both divide by 1, but* $5x$ *shouldn’t have a* $-$*, so I’ll use* $-1$$$ *&* $$ |
| ***Third Step:*** *Factor what you thought of in the thinking step out of each group.* | $$f\left(x\right)=3[5x^{2}+1x-5x-1]$$$$f\left(x\right)=3[x(5x+1)-1(5x+1)]$$ |
| ***Fourth Step:*** *Factor out the ( ) that they both have in common.* | $$f\left(x\right)=3(5x+1)(x-1)$$*The factored form of* $f\left(x\right)=15x^{2}-12x-3$ *is*$$$$ |
| *We have factored form! Now, let’s find the roots!* |
| ***Fifth Step:*** *Set each factor involving x equal to zero, and solve for x.* | $$f\left(x\right)=3(5x+1)(x-1)$$ |
| $\left(5x+1\right)=0$$$5x=-1$$$$x=-\frac{1}{5}$$ | $$\left(x-1\right)=0$$$$x=1$$ |
| ***Sixth Step:*** *Write your roots as points.* | *The roots of* $f\left(x\right)=15x^{2}-12x-3$ *are* $\left(-\frac{1}{5}, 0\right)\& (1, 0)$*.* |

For each problem, write the polynomial in factored form, and then determine the roots.

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| **EXAMPLE:** $f\left(x\right)=-x^{2}+8x+20$$$f\left(x\right)=-x^{2}+8x+20$$$$f\left(x\right)=-[x^{2}-8x-20]$$

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| $$A=+1$$ | $$B=-8$$ | $$C=-20$$ |
| $$AC=$$$$\left(1\right)\left(-20\right)$$$$-20$$ | Multiply to AC, Combine to B |
| $$\left(2\right)\left(-10\right)=-20=AC$$& $2-10=-8=B$So… $2 \&-10$! |

$f\left(x\right)=-[x^{2}-8x-20]$ ***Split -8x***$$f\left(x\right)=-[x^{2}+2x-10x-20]$$$$f\left(x\right)=-\left[x\left(x+2\right)-10\left(x+2\right)\right]$$$$$$ | **EXAMPLE:** $f\left(x\right)=4x^{2}-6x+2$$$f\left(x\right)=4x^{2}-6x+2$$$$f\left(x\right)=2[2x^{2}-3x+1]$$

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| --- | --- | --- |
| $$A=2$$ | $$B=-3$$ | $$C=1$$ |
| $$AC=$$$$\left(2\right)\left(1\right)$$$$2$$ | Multiply to AC, Combine to B |
| $$\left(-2\right)\left(-1\right)=2=AC$$& $-2-1=-3=B$So… $-2 \&-1$! |

$$f\left(x\right)=2[2x^{2}-2x-1x+1]$$$$f\left(x\right)=2[2x\left(x-1\right)-1\left(x-1\right)]$$$$$$ | 1. $g\left(x\right)=3x^{2}+9x-30$

$$g\left(x\right)=3[ ]$$

|  |  |  |
| --- | --- | --- |
| $$A=$$ | $$B=$$ | $$C=$$ |
| $$AC=$$ | Multiply to AC, Combine to B |
|  |

 |
| *The roots are:* | $$x-1=0$$$$x=1$$$$$$ | $$2x-1=0$$$$2x=1$$$x=\frac{1}{2}$ $$  |
| *The roots are:* | $$(x+2)=0$$$$x=-2$$$$$$ | $$(x-10)=0$$$$x=10$$$$$$ |
| 1. $h\left(x\right)=-2x^{2}+7x-6$

$$h\left(x\right)=-[ ]$$

|  |  |  |
| --- | --- | --- |
| $$A=$$ | $$B=$$ | $$C=$$ |
| $$AC=$$ | Multiply to AC, Combine to B |
|  |

*Set each root = to 0 & solve for* x*.* | 1. $k\left(x\right)=-6x^{2}-28x+10$

$$k\left(x\right)=-2[ ]$$

|  |  |  |
| --- | --- | --- |
| $$A=$$ | $$B=$$ | $$C=$$ |
| $$AC=$$ | Multiply to AC, Combine to B |
|  |

*Set each root = to 0 & solve for* x*.* | 1. $m\left(x\right)=12x^{2}-75$

$$\left(Insert 0x\right) m\left(x\right)=12x^{2}+0x-27$$$$k\left(x\right)=3[ ]$$

|  |  |  |
| --- | --- | --- |
| $$A=$$ | $$B=$$ | $$C=$$ |
| $$AC=$$ | Multiply to AC, Combine to B |
|  |

 |
| 1. $n\left(x\right)=-7x^{2}+7$

$$\left(Insert 0x\right)$$$$n\left(x\right)=\\_\\_\\_[ ]$$

|  |  |  |
| --- | --- | --- |
| $$A=$$ | $$B=$$ | $$C=$$ |
| $$AC=$$ | Multiply to AC, Combine to B |
|  |

 | 1. $p\left(x\right)=90x^{2}-160$

|  |  |  |
| --- | --- | --- |
| $$A=$$ | $$B=$$ | $$C=$$ |
| $$AC=$$ | Multiply to AC, Combine to B |
|  |

 | 1. $r\left(x\right)=5x^{2}-13x-6$

|  |  |  |
| --- | --- | --- |
| $$A=$$ | $$B=$$ | $$C=$$ |
| $$AC=$$ | Multiply to AC, Combine to B |
|  |

 |

When you’re done solving the problems on this handout, make sure you check your answers

against the **correct answers below**.

|  |  |  |
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| 1. $–x^{2}+7x+4$
 | 1. $2x^{3}+x^{2}+3x-8$
 | 1. $-5x^{2}+4x+9$
 |
| 1. $3x^{3}-5x-1$
 | 1. $5x^{2}+6x-14$
 | 1. $7x$
 |
| 1. $12x^{2}-4x-5$
 | 1. $12x^{2}+14x-10$
 | 1. $-21x^{2}+53x-30$
 |
| 1. $28x^{3}+8x^{2}-20x$
 | 1. $–x^{2}+3x+18$
 | 1. $9x^{3}-63x^{2}-6x+42$
 |
| 1. $12x^{2}+54x+54$
 | 1. $40x^{2}+31x-6$
 | 1. Factored Form:

$$g\left(x\right)=3(x-2)(x+5)$$Roots:$ \left(2, 0\right)\&(-5, 0)$ |
| 1. Factored Form:

$$h\left(x\right)=-(2x-3)(x-2)$$Roots:$ \left(\frac{3}{2}, 0\right)\&(2, 0)$ | 1. Factored Form:

$$k\left(x\right)=-2(3x-1)(x+5)$$Roots:$ \left(\frac{1}{3}, 0\right)\&(-5, 0)$ | 1. Factored Form:

$$m\left(x\right)=3(2x+5)(2x-5)$$Roots:$ \left(-\frac{5}{2}, 0\right)\&\left(\frac{5}{2}, 0\right)$ |
| 1. Factored Form:

$$n\left(x\right)=-7(x+1)(x-1)$$Roots:$ \left(-1, 0\right)\&(1, 0)$ | 1. Factored Form:

$$p\left(x\right)=10(3x+4)(3x-4)$$Roots:$ \left(-\frac{4}{3}, 0\right)\&\left(\frac{4}{3}, 0\right)$ | 1. Factored Form:

$$r\left(x\right)=(5x+2)(x-3)$$Roots:$ \left(-\frac{2}{5}, 0\right)\&(3, 0)$ |