Graphing & Writing Solutions to Quadratic Inequalities

For each quadratic, I have solved for the vertex, y-intercept and zeros. Your job is to graph the equation on the right, and then, below, to graph each different type of inequality. Remember:

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| If you have $ $$$ f\left(x\right)<quadratic $$or you have $quadratic>f(x)$ | If you have $ $$$ f\left(x\right)\leq quadratic $$or you have $quadratic\geq f(x)$ | If you have $ $$$ f\left(x\right)>quadratic $$or you have $quadratic<f(x)$ | If you have $ $$$ f\left(x\right)\geq quadratic $$or you have $quadratic\leq f(x)$ |
| No “or equal” means the curve is DOTTED\_\_\_ \_\_\_ \_\_\_ \_\_\_ \_\_\_*y* is less than, so we will shade on the side of curve that is below the vertex | “Or equal” makes the curve SOLID\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_*y* is less than, so we will shade on the side of curve that is below the vertex | No “or equal” means the curve is DOTTED\_\_\_ \_\_\_ \_\_\_ \_\_\_ \_\_\_*y* is greater than, so we shade on the side of curve that is above the vertex | “Or equal” makes the curve SOLID\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_*y* is greater than, so we shade on the side of curve that is above the vertex |
| If the shaded area is the **inside** of the parabola, the solutions will be:$$x\in \left(lower zero, upper\right)$$ | If the shaded area is the **inside** of the parabola, the solutions will be:$$x\in [lower zero, upper]$$ | If the shaded area is the **inside** of the parabola, the solutions will be:$$x\in \left(lower zero, upper\right)$$ | If the shaded area is the **inside** of the parabola, the solutions will be:$$x\in [lower zero, upper]$$ |
| If the shaded area is the **outside** of the parabola, the solutions will be:$x\in \left(-\infty , lower zero\right)$ *or*$$\left(upper zero, \infty \right)$$ | If the shaded area is the **outside** of the parabola, the solutions will be:$x\in (-\infty , lower zero]$ *or*$$[upper zero, \infty )$$ | If the shaded area is the **outside** of the parabola, the solutions will be:$x\in \left(-\infty , lower zero\right)$ *or*$$\left(upper zero, \infty \right)$$ | If the shaded area is the **outside** of the parabola, the solutions will be:$x\in (-\infty , lower zero]$ *or*$$[upper zero, \infty )$$ |

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| **1.** $f\left(x\right)=x^{2}+4x-5$ | $$y-intercept$$$$ x=0$$$$f\left(0\right)=\left(0\right)^{2}+4\left(0\right)-5$$$$f\left(0\right)=0+0-5$$$$f\left(0\right)=-5$$$$y-int:$$ | $$a=1, b=4, c=-5$$Zeros:$$x=\frac{-b\pm \sqrt{b^{2}-4ac}}{2a}$$$$x=\frac{-(4)\pm \sqrt{(4)^{2}-4(1)(-5)}}{2(1)}$$$$x=\frac{-4\pm \sqrt{16+20}}{2}=\frac{-4\pm \sqrt{36}}{2}$$$$x=\frac{-4+6}{2}=\frac{2}{2}=1 $$$$or x=\frac{-4-6}{2}=\frac{-10}{2}=-5 $$Zeros: $$ | $$f\left(x\right)=x^{2}+4x-5$$ |
| Vertex in standard form:$$\left(\frac{-b}{2a}, plug it in\right)$$$$x=\frac{-(4)}{2\left(1\right)}=\frac{-4}{2}=-2$$$$y=\left(-2\right)^{2}+4\left(-2\right)-5$$$$y=4-8-5$$$$y=-4-5$$$$y=-9$$Vertex: $$ |  |
| $$f\left(x\right)<x^{2}+4x-5$$ | $$f\left(x\right)\leq x^{2}+4x-5$$ | $$f\left(x\right)>x^{2}+4x-5$$ | $$f\left(x\right)\geq x^{2}+4x-5$$ |
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| **2.** $f\left(x\right)=-x^{2}+2x+3$ | $$y-intercept$$$$ x=0$$$$f\left(0\right)=-\left(0\right)^{2}+2\left(0\right)+3$$$$f\left(0\right)=-0+0+3$$$$f\left(0\right)=3$$$$y-int:$$ | $$a=-1, b=2, c=3$$Zeros:$$x=\frac{-b\pm \sqrt{b^{2}-4ac}}{2a}$$$$x=\frac{-(2)\pm \sqrt{(2)^{2}-4(-1)(3)}}{2(-1)}$$$$x=\frac{-2\pm \sqrt{4+12}}{-2}=\frac{-2\pm \sqrt{16}}{-2}$$$$x=\frac{-2+4}{-2}=\frac{2}{-2}=-1 $$$$or x=\frac{-2-4}{-2}=\frac{-6}{-2}=3 $$Zeros: $$ | $$f\left(x\right)=-x^{2}+2x+3$$ |
| Vertex in standard form:$$\left(\frac{-b}{2a}, plug it in\right)$$$$x=\frac{-(2)}{2\left(-1\right)}=\frac{-2}{-2}=1$$$$y=-\left(1\right)^{2}+2\left(1\right)+3$$$$y=-\left(1\right)+2+3$$$$y=1+3$$$$y=4$$Vertex: $$ |  |
| $$f\left(x\right)<-x^{2}+2x+3$$ | $$f\left(x\right)\leq -x^{2}+2x+3$$ | $$f\left(x\right)>-x^{2}+2x+3$$ | $$f\left(x\right)\geq -x^{2}+2x+3$$ |
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| **3.** $f\left(x\right)=2x^{2}-8x+8$ | $$y-intercept$$$$ x=0$$$$f\left(0\right)=2\left(0\right)^{2}-8\left(0\right)+8$$$$f\left(0\right)=0+0+8$$$$f\left(0\right)=8$$$$y-int:$$ | $$a=2, b=-8, c=8$$Zeros:$$x=\frac{-b\pm \sqrt{b^{2}-4ac}}{2a}$$$$x=\frac{-(-8)\pm \sqrt{(-8)^{2}-4(2)(8)}}{2(2)}$$$$x=\frac{8\pm \sqrt{64-64}}{4}=\frac{8\pm \sqrt{0}}{4}$$$$x=\frac{8+0}{4}=\frac{8}{4}=2 $$$$or x=\frac{8-0}{4}=\frac{8}{4}=2 $$Zeros: $$ *It’s a double root.* | $$f\left(x\right)=2x^{2}-8x+8$$ |
| Vertex in standard form:$$\left(\frac{-b}{2a}, plug it in\right)$$$$x=\frac{-(-8)}{2\left(2\right)}=\frac{8}{4}=2$$$$y=2\left(2\right)^{2}-8\left(2\right)+8$$$$y=2\left(4\right)-16+8$$$$y=8-16+8$$$$y=-8+8$$$$y=0$$Vertex: $$ |  |
| $$f\left(x\right)<2x^{2}-8x+8$$ | $$f\left(x\right)\leq 2x^{2}-8x+8$$ | $$f\left(x\right)>2x^{2}-8x+8$$ | $$f\left(x\right)\geq 2x^{2}-8x+8$$ |
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| **4.** $f\left(x\right)=-x^{2}-5x-4$ | $$y-intercept$$$$ x=0$$$f\left(0\right)$ =$ -\left(0\right)^{2}-5\left(0\right)-4$$$f\left(0\right)=-0-0-4$$$$f\left(0\right)=-4$$$$y-int:$$ | $$a=-1, b=-5, c=-4$$Zeros:$$x=\frac{-b\pm \sqrt{b^{2}-4ac}}{2a}$$$$x=\frac{-(-5)\pm \sqrt{(-5)^{2}-4(-1)(-4)}}{2(-1)}$$$$x=\frac{5\pm \sqrt{25-16}}{-2}=\frac{5\pm \sqrt{9}}{-2}$$$$x=\frac{5+3}{-2}=\frac{8}{-2}=-4 $$$$or x=\frac{5-3}{-2}=\frac{2}{-2}=-1 $$Zeros: $$ | $$f\left(x\right)=-x^{2}-5x-4$$ |
| Vertex in standard form:$$\left(\frac{-b}{2a}, plug it in\right)$$$$x=\frac{-\left(-5\right)}{2\left(-1\right)}=\frac{5}{-2}$$$$x=-2.5$$$$y=-\left(-2.5\right)^{2}-5\left(-2.5\right)-4$$$$y=-6.25+12.5-4$$$$y=6.25-4$$$$y=2.25$$Vertex: $$ |  |
| $$f\left(x\right)<-x^{2}-5x-4$$ | $$f\left(x\right)\leq -x^{2}-5x-4$$ | $$f\left(x\right)>-x^{2}-5x-4$$ | $$f\left(x\right)\geq -x^{2}-5x-4$$ |
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| **5.** $f\left(x\right)=\left(x+3\right)^{2}-1$ | $y-intercept$:$$ x=0$$$$f\left(0\right)=\left(0+3\right)^{2}-1$$$$f\left(0\right)=\left(3\right)^{2}-1$$$$f\left(0\right)=9-1$$$$f\left(0\right)=8$$$$y-int:$$ | *not in standard form* $\rightarrow $ *just set = 0!*Zeros: $$f\left(x\right)=\left(x+3\right)^{2}-1$$$$ \left(x+3\right)^{2}-1=0$$$ \left(x+3\right)^{2}-1+1=0+1$ $$ \left(x+3\right)^{2}=1$$$$ \sqrt{\left(x+3\right)^{2}}=\pm \sqrt{1}$$$$ x+3=\pm 1$$$$ x+3-3=-3\pm 1$$$x=-3+ 1$ or $x=-3- 1$$x=-2$ $x=-4$Zeros: $$ | $$f\left(x\right)=\left(x+3\right)^{2}-1$$ |
| Vertex in vertex form:*(Vertex is in the equation)*$$a\left(x\pm OPP.x\right)^{2}\pm SAMEy$$$$\left(x\right)^{2}$$Vertex: $$ |  |
| $$f\left(x\right)<\left(x+3\right)^{2}-1$$ | $$f\left(x\right)\leq \left(x+3\right)^{2}-1$$ | $$f\left(x\right)>\left(x+3\right)^{2}-1$$ | $$f\left(x\right)\geq \left(x+3\right)^{2}-1$$ |
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| **6.** $f\left(x\right)=-3\left(x-2\right)^{2}-9$ | $$y-intercept$$$$ x=0$$$$f\left(0\right)=-3\left(0-2\right)^{2}-9$$$$f\left(0\right)=-3\left(2\right)^{2}-9$$$$f\left(0\right)=-3\left(4\right)-9$$$$f\left(0\right)=-12-9$$$$f\left(0\right)=-21$$$$y-int:$$ | *not in standard form* $\rightarrow $ *just set = 0!*Zeros: $$f\left(x\right)=-3\left(x-2\right)^{2}-9$$$$ -3\left(x-2\right)^{2}-9=0$$$ -3\left(x-2\right)^{2}-9+9=0+9$ $$ -3\left(x-2\right)^{2}=9$$ $÷-3 ÷-3$$$ \left(x-2\right)^{2}=-3$$$$ \sqrt{\left(x-2\right)^{2}}=\pm \sqrt{-3}$$$$ x-2=\pm \sqrt{-3}$$$$ x-2+2=+2\pm \sqrt{-3}$$$x=2+\sqrt{-3}$ or $x=2-\sqrt{-3}$Zeros: $$ | $$f\left(x\right)=-3\left(x-2\right)^{2}-9$$ |
| Vertex in vertex form:*(Vertex is in the equation)*$$a\left(x\pm OPP.x\right)^{2}\pm SAMEy$$$$-3\left(x\right)^{2}$$Vertex: $$ |  |
| $$f\left(x\right)<-3\left(x-2\right)^{2}-9$$ | $$f\left(x\right)\leq -3\left(x-2\right)^{2}-9$$ | $$f\left(x\right)>-3\left(x-2\right)^{2}-9$$ | $$f\left(x\right)\geq -3\left(x-2\right)^{2}-9$$ |
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| **7.** $f\left(x\right)=4\left(x+1\right)^{2}-8$ | $$y-intercept$$$$ x=0$$$$f\left(0\right)=4\left(0+1\right)^{2}-8$$$$f\left(0\right)=4\left(1\right)^{2}-8 $$$$f\left(0\right)=4\left(1\right)-8$$$$f\left(0\right)=4-8$$$$f\left(0\right)=-4$$$$y-int:$$ | *not in standard form* $\rightarrow $ *just set = 0!*Zeros: $$f\left(x\right)=4\left(x+1\right)^{2}-8$$$$ 4\left(x+1\right)^{2}-8=0$$$ 4\left(x+1\right)^{2}-8+8=0+8$ $$ 4\left(x+1\right)^{2}=8$$ $÷4 ÷4$$$ \left(x+1\right)^{2}=2$$$$ \sqrt{\left(x+1\right)^{2}}=\pm \sqrt{2}$$$$ x+1=\pm \sqrt{2}$$$$ x+1-1=-1\pm \sqrt{2}$$$x=-1+\sqrt{2}$ or $x=-1-\sqrt{2}$Zeros: $$ | $$f\left(x\right)=4\left(x+1\right)^{2}-8$$ |
| Vertex in vertex form:*(Vertex is in the equation)*$$a\left(x\pm OPP.x\right)^{2}\pm SAMEy$$$$4\left(x\right)^{2}$$Vertex: $$ |  |
| $$f\left(x\right)<4\left(x+1\right)^{2}-8$$ | $$f\left(x\right)\leq 4\left(x+1\right)^{2}-8$$ | $$f\left(x\right)>4\left(x+1\right)^{2}-8$$ | $$f\left(x\right)\geq 4\left(x+1\right)^{2}-8$$ |
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