Imaginary Numbers and the Quadratic Formula

In the real world, it is not possible to square root a negative number. However, higher math often depends on the ability to do so. This is where *imaginary numbers* $(i)$ come into play. Whenever you see a negative inside of a radical, just remember that $\sqrt{-1}=i$, so you can take out the negative by putting an $i$ out front.

Simplify the radicals.

|  |  |  |  |
| --- | --- | --- | --- |
| **EXAMPLE**$$\sqrt{45} =\sqrt{9}\sqrt{5}$$$$ =$$ | **EXAMPLE**$\sqrt{-45}$ $=\sqrt{9}\sqrt{-1}\sqrt{5}$$$ =$$ | **EXAMPLE**$$\sqrt{28} =\sqrt{4}\sqrt{7}$$$$ =$$ | **EXAMPLE**$$\sqrt{-28} =\sqrt{4}\sqrt{-1}\sqrt{7}$$$$ =$$ |
| 1. $\sqrt{20}$ | 2. $\sqrt{-20}$ | 3. $\sqrt{125}$ | 4. $\sqrt{-125}$ |
| 5. $\sqrt{36}$ | 6. $\sqrt{-36}$ | 7. $\sqrt{98}$ | 8. $\sqrt{-98}$ |
| 9. $\sqrt{128}$ | 10. $\sqrt{-128}$ | 11. $\sqrt{121}$ | 12. $\sqrt{-121}$ |

Determine the *solutions* to the quadratics below using the quadratic formula (shown below). Remember, whenever you have the square root of a negative number, you have to take the negative out and make it an $i$.

$$x=\frac{-b\pm \sqrt{b^{2}-4ac}}{2a}$$

|  |  |
| --- | --- |
| 13. Solve $f\left(x\right)=-3x^{2}+4x-6$. | 14. Solve $f\left(x\right)=2x^{2}-4x+2$ |
| 15. Solve $f\left(x\right)=2x^{2}+8x+8$. | 16. Solve $f\left(x\right)=x^{2}+4x+4$. |
| 17. Solve $f\left(x\right)=-x^{2}-8x-19$. | 18. Solve $f\left(x\right)=-3x^{2}-6x+9$. |

So far, we have worked with negative radicals, turning them into imaginary numbers $(i)$. The next piece is knowing what happens when you raise an imaginary number to a power (use an exponent on it). This is where I use what I call the circle of $i$.



So, according to the circle above, $i^{0}=1, i^{1}=i, i^{2}=-1, \& i^{3}=-i$. If I kept going around the circle as my exponent got bigger, I would know how to simplify my imaginary number. Go around the circle to simplify the imaginary numbers below.

|  |  |  |  |
| --- | --- | --- | --- |
| 19. $i^{0}$ | 20. $i^{1}$ | 21. $i^{2}$ | 22. $i^{3}$ |
| 23. $i^{4}$ | 24. $i^{5}$ | 25. $i^{6}$ | 26. $i^{7}$ |
| 27. $i^{8}$ | 28. $i^{9}$ | 29. $i^{10}$ | 30. $i^{11}$ |
| 31. $i^{12}$ | 32. $i^{13}$ | 33. $i^{14}$ | 34. $i^{15}$ |

So far, they’ve all been in order. Now, we’re going to mix things up a bit. Don’t worry, though—I’ll give you a trick to make things easier.

$i^{4}=1 $and if you multiply by 1 or $1^{2}$ or$ 1^{307}$, thn you’re not changing anything. $i^{4}$ doesn’t change the answer.

THIS MEANS…

 You can always get rid of $i^{4}$ —whatever REMAINS is the problem you’re actually solving. The $i^{4}$ is just extra.

So, here's **the trick**:

 Divide the exponent by 4.

The REMAINDER is the only part of the exponent you need. The rest can just be canceled out.

|  |  |  |  |
| --- | --- | --- | --- |
| **EXAMPLE**$$i^{282}= ?$$ |  | $i^{282}=i^{280}i^{2}$$$i^{282}=\left(i^{4}\right)^{70}i^{2}$$$i^{282}=(1)^{70}\left(\sqrt{-1}\right)^{2}$$$i^{282}=$$ | **Same problem…the short version:** |
| **EXAMPLE**$$i^{282}= ?$$ |  | *Ignore everything except the remainder…*$$i^{282}= i^{2}=$$ |

|  |  |  |  |
| --- | --- | --- | --- |
| **EXAMPLE**$$i^{51}= ?$$$$i^{51}=i^{3}$$$$i^{3}=\sqrt{-1}\sqrt{-1}\sqrt{-1}$$$$i^{3}=\left(-1\right)\sqrt{-1}$$$$i^{3}=$$ | 35. $i^{47}= ?$ | 36. $i^{26}= ?$ | 37. $i^{111}= ?$ |
| **EXAMPLE**$$i^{576}= ?$$$$i^{576}=i^{0}$$$$i^{0}=$$ | 38. $i^{205}= ?$ | 39. $i^{342}= ?$ | 40. $i^{19}= ?$ |