Name: ____________________________

Completing the Square (Converting Standard to Vertex form)

The solutions to a quadratic tell us where the parabola crosses the *x*-axis (where $y = 0$). So far, we have only used the quadratic formula, but there are multiple ways to solve a quadratic. In vertex form, all you have to do is set the problem equal to zero (because the solutions to a quadratic are where the graph crosses the *x*-axis at $y = 0$) and solve.

Step 1: Set it equal to Zero Step 2: Add and divide to get the $(\_\_)^2$ alone

Step 3: Square root both sides. (Don't forget: $\pm\sqrt{\_\_} = \pm\sqrt{(\_\_)^2}$) Step 4: Solve it.

EXAMPLE $f(x) = 5(x+2)^2 - 45$ Step 1: $= 0$ $5(x+2)^2 - 45 = 0$ Step 2: Isolate $(\quad)^2$ $5(x+2)^2 - 45 = 0$ $\underline{+45 = +45}$ $5(x+2)^2 = 45$ $\underline{\div 5} \quad = \underline{\div 5}$ $(x+2)^2 = 9$ Step 3: $\sqrt{(\quad)^2} = \pm\sqrt{\#}$ $\sqrt{(x+2)^2} = \pm\sqrt{9}$ $x + 2 = \pm 3$ Step 4: **Solve for the roots.** $x + 2 = +3$ *or* $x + 2 = -3$ $x = -2 + 3 \quad x = -2 - 3$ $x = \boxed{1}$ *or* $x = \boxed{-5}$	1. $f(x) = 2(x+6)^2 - 8$	2. $f(x) = 4(x+1)^2 - 36$	3. $f(x) = 8(x-7)^2 - 8$
EXAMPLE $f(x) = 3(x-4)^2 - 36$ Step 1: $= 0$ $3(x-4)^2 - 36 = 0$ Step 2: Isolate $(\quad)^2$ $3(x-4)^2 - 36 = 0$ $\underline{+36 = +36}$ $3(x-4)^2 = 36$ $\underline{\div 3} \quad = \underline{\div 3}$ $(x-4)^2 = 12$ Step 3: $\sqrt{(\quad)^2} = \pm\sqrt{\#}$ $\sqrt{(x-4)^2} = \pm\sqrt{12}$ $x - 4 = \pm\sqrt{4}\sqrt{3}$ $x - 4 = \pm 2\sqrt{3}$ Step 4: **Solve for the roots.** $x - 4 = +2\sqrt{3}$ *or* $x - 4 = -2\sqrt{3}$ $x = \boxed{4 + 2\sqrt{3}} \quad x = \boxed{4 - 2\sqrt{3}}$	4. $f(x) = -3(x-5)^2 + 18$	5. $f(x) = -(x+9)^2 + 27$	6. $f(x) = -5(x-3)^2 - 35$
7. $f(x) = 2(x-1)^2$	8. $f(x) = (x+4)^2 + 1$	9. $f(x) = -3(x+5)^2$	10. $f(x) = 2(x)^2 - 8$

If you want to solve a quadratic this way, but you have the problem in *standard form*, not vertex form (as shown above), then you have to do something called completing the square. The steps, if you follow them, are easy enough, but you have to follow them (no matter how strange they seem).

EXAMPLE

$f(x) = 5x^2 + 20x - 25$

$5x^2 + 20x - 25 \boxed{= 0}$	Step 1: Lose $f(x)$ & set it equal to 0.
$5x^2 + 20x = +25$	Step 2: Kick out c (move it to the other side of =)
$5(x^2 + 4x) = +25$	Step 3: Factor a out to make $a(\ ) = \#$.
$5(x^2 + 4x) = +25$ $b = 4$ $5\left(x^2 + 4x \boxed{+\left(\frac{4}{2}\right)^2}\right) = +25 + 5\left(\boxed{+\left(\frac{4}{2}\right)^2}\right)$	Step 4: Using the new b inside $a(\ )$, add $\left(\frac{b}{2}\right)^2$ inside ( ) and add $a\left(\frac{b}{2}\right)^2$ to the other side of =
$5\left(x^2 + 4x + \boxed{(2)^2}\right) = +25 + 5\boxed{(2)^2}$ $5(x^2 + 4x + (2)^2) = +25 + \boxed{5(4)}$ $5(x^2 + 4x + (2)^2) = +25\boxed{+20}$ $5(x^2 + 4x + (2)^2) = \boxed{45}$	Step 5: Simplify only the $\frac{b}{2}$ part inside & all of the other side
$5\left(\boxed{x}^2 + 4x + \left(\boxed{2}\right)^2\right) = 45$ $5\left(\boxed{x}\boxed{+2}\right)^2 = 45$	Step 6: *MAGIC FACTOR* Change the *entire parenthesis* to: $\left(x + \frac{b}{2}\right)^2$ ...trust me.
$5(x + 2)^2 = 45$ $\div 5 \qquad \div 5$ $(x + 2)^2 = 9$ $\sqrt{(x + 2)^2} = \pm\sqrt{9}$ $x + 2 = \pm 3$ $x + 2 = +3 \quad or \quad x + 2 = -3$ $x = -2 + 3 \qquad x = -2 - 3$ $x = \boxed{1} \qquad x = \boxed{-5}$	Step 7: Now, you can solve it!

10. $f(x) = 4x^2 + 4x + 40$

	Step 1: Lose $f(x)$ & set it equal to 0.
	Step 2: Kick out c (move it to the other side of =)
	Step 3: Factor a out of the parentheses.
	Step 4: Using the new b inside $a(\ )$, add $\left(\frac{b}{2}\right)^2$ inside ( ) and add $a\left(\frac{b}{2}\right)^2$ to the other side of =
	Step 5: Simplify only the $\frac{b}{2}$ part inside & all of the other side
	Step 6: *MAGIC FACTOR* Change the *entire parenthesis* to: $\left(x + \frac{b}{2}\right)^2$ ...trust me.
	Step 7: Now, you can solve it!

11. $f(x) = 3x^2 - 24x + 84$

	Step 1: Lose $f(x)$ & set it equal to 0.
	Step 2: Kick out c (move it to the other side of =)
	Step 3: Factor a out of the parentheses.
	Step 4: Using the <u>new</u> b inside $a(\ )$, add $\left(\frac{b}{2}\right)^2$ inside ( ) and add $a\left(\frac{b}{2}\right)^2$ to the other side of =
	Step 5: Simplify <u>only</u> the $\frac{b}{2}$ part inside & <u>all</u> of the other side
	Step 6: *MAGIC FACTOR* Change the *entire parenthesis* to: $\left(x + \frac{b}{2}\right)^2$ ...trust me.
	Step 7: Now, you can solve it!

12. $f(x) = 2x^2 + 24x + 56$

	Step 1: Lose $f(x)$ & set it equal to 0.
	Step 2: Kick out c (move it to the other side of =)
	Step 3: Factor a out of the parentheses.
	Step 4: Using the <u>new</u> b inside $a(\ )$, add $\left(\frac{b}{2}\right)^2$ inside ( ) and add $a\left(\frac{b}{2}\right)^2$ to the other side of =
	Step 5: Simplify <u>only</u> the $\frac{b}{2}$ part inside & <u>all</u> of the other side
	Step 6: *MAGIC FACTOR* Change the *entire parenthesis* to: $\left(x + \frac{b}{2}\right)^2$ ...trust me.
	Step 7: Now, you can solve it!

13. $f(x) = 5x^2 - 30x - 50$

	Step 1: Lose $f(x)$ & set it equal to 0.
	Step 2: Kick out c (move it to the other side of =)
	Step 3: Factor a out of the parentheses.
	Step 4: Using the new b inside $a(\ )$, add $\left(\frac{b}{2}\right)^2$ inside ( ) and add $a\left(\frac{b}{2}\right)^2$ to the other side of =
	Step 5: Simplify only the $\frac{b}{2}$ part inside & all of the other side
	Step 6: *MAGIC FACTOR* Change the *entire parenthesis* to: $\left(x + \frac{b}{2}\right)^2$ ...trust me.
	Step 7: Now, you can solve it!

14. $f(x) = -x^2 - 18x - 54$

	Step 1: Lose $f(x)$ & set it equal to 0.
	Step 2: Kick out c (move it to the other side of =)
	Step 3: Factor a out of the parentheses.
	Step 4: Using the new b inside $a(\ )$, add $\left(\frac{b}{2}\right)^2$ inside ( ) and add $a\left(\frac{b}{2}\right)^2$ to the other side of =
	Step 5: Simplify only the $\frac{b}{2}$ part inside & all of the other side
	Step 6: *MAGIC FACTOR* Change the *entire parenthesis* to: $\left(x + \frac{b}{2}\right)^2$ ...trust me.
	Step 7: Now, you can solve it!