Name: Solution Guide Per: ____

Integrated II: Unit 2 Study Guide

1. Find the value of r.

$(r+4)^2 = 48$

$\pm\sqrt{(r+4)^2} = \pm\sqrt{48}$

$r+4 = \pm\sqrt{16}\sqrt{3}$

$r+4 = \pm 4\sqrt{3}$

$-4 \quad -4$

$r = \boxed{-4 \pm 4\sqrt{3}}$

2. Find the value of s.

$(s-2)^2 = 200$

$\pm\sqrt{(s-2)^2} = \pm\sqrt{200}$

$s-2 = \pm\sqrt{100}\sqrt{2}$

$s-2 = \pm 10\sqrt{2}$

$+2 \quad +2$

$s = \boxed{2 \pm 10\sqrt{2}}$

3. Find the value of d.

$(d+3)^2 = 12$

$\pm\sqrt{(d+3)^2} = \pm\sqrt{12}$

$d+3 = \pm\sqrt{4}\sqrt{3}$

$d+3 = \pm 2\sqrt{3}$

$-3 \quad -3$

$d = -3 \pm 2\sqrt{3}$

4. Determine the roots of the equation $x^2 + 5x - 36 = 0$.

$x = \frac{-5 \pm \sqrt{(5)^2 - 4(1)(-36)}}{2(1)}$

$x = \frac{-5 \pm \sqrt{25+144}}{2}$

$x = \frac{-5 \pm \sqrt{169}}{2} = \frac{-5 \pm 13}{2}$

$x = \frac{-5+13}{2}$ or $x = \frac{-5-13}{2}$

$x = \frac{8}{2} = 4$ or $x = \frac{-18}{2} = -9$

5. Determine the roots of the equation $x^2 - x - 42 = 0$.

$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-42)}}{2(1)}$

$x = \frac{1 \pm \sqrt{1+168}}{2} = \frac{1 \pm \sqrt{169}}{2}$

$x = \frac{1 \pm 13}{2} \Rightarrow \frac{1+13}{2} = \frac{14}{2} = 7$

or $\frac{1-13}{2} = \frac{-12}{2} = -6$

$x = \boxed{-6 \text{ or } 7}$

6. Determine the roots of the equation $x^2 - 5x - 14 = 0$.

$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(-14)}}{2(1)}$

$x = \frac{5 \pm \sqrt{25+56}}{2}$

$x = \frac{5 \pm \sqrt{81}}{2} = \frac{5 \pm 9}{2} \Rightarrow \frac{5+9}{2} = \frac{14}{2} = 7$

$\Rightarrow \frac{5-9}{2} = \frac{-4}{2} = -2$

$x = \boxed{-2 \text{ or } 7}$

7. Calculate the roots of the quadratic equation, if possible. Verify your solution.

$3x^2 + 10x + 8 = 0$

$x = \frac{-10 \pm \sqrt{(10)^2 - 4(3)(8)}}{2(3)}$

$x = \frac{-10 \pm \sqrt{100-96}}{6} = \frac{-10 \pm \sqrt{4}}{6}$

$x = \frac{-10 \pm 2}{6} \Rightarrow \frac{-10+2}{6} = \frac{-8}{6} = -\frac{4}{3}$

$\Rightarrow \frac{-10-2}{6} = \frac{-12}{6} = -2$

$x = \boxed{-\frac{4}{3} \text{ or } -2}$

8. Calculate the roots of the quadratic equation, if possible. Verify your solution.

$4x^2 + 10x + 6 = 0$

$x = \frac{-10 \pm \sqrt{(10)^2 - 4(4)(6)}}{2(4)}$

$x = \frac{-10 \pm \sqrt{100-96}}{8} = \frac{-10 \pm \sqrt{4}}{8}$

$x = \frac{-10 \pm 2}{8} \Rightarrow \frac{-10+2}{8} = \frac{-8}{8} = -1$

$\Rightarrow \frac{-10-2}{8} = \frac{-12}{8} = -\frac{3}{2}$

$x = \boxed{-1 \text{ or } -\frac{3}{2}}$

9. Calculate the roots of the quadratic equation, if possible. Verify your solution.

$5x^2 + 7x - 6 = 0$

$x = \frac{-7 \pm \sqrt{(7)^2 - 4(5)(-6)}}{2(5)}$

$x = \frac{-7 \pm \sqrt{49+120}}{10} = \frac{-7 \pm \sqrt{169}}{10}$

$x = \frac{-7 \pm 13}{10} \Rightarrow \frac{-7+13}{10} = \frac{6}{10} = \frac{3}{5}$

$\Rightarrow \frac{-7-13}{10} = \frac{-20}{10} = -2$

$x = \boxed{\frac{3}{5} \text{ or } -2}$

Name: _____ Per: ____

10. Use the quadratic formula to find the zeros.

$f(x) = 2x^2 + 4x - 3$

$x = \frac{-4 \pm \sqrt{16 - 4(2)(-3)}}{4} = \frac{-4 \pm \sqrt{40}}{4} = \frac{-4 \pm 2\sqrt{10}}{4}$

$x = \frac{-4 \pm 2\sqrt{10}}{4} = \frac{-2 \pm \sqrt{10}}{2}$

$x = \frac{-2}{2} \pm \frac{\sqrt{10}}{2} = -1 \pm \frac{\sqrt{10}}{2}$

11. Use the quadratic formula to find the zeros.

$f(x) = x^2 - 8x + 1$

$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(1)(1)}}{2(1)}$

$x = \frac{8 \pm \sqrt{64 - 4}}{2} = \frac{8 \pm \sqrt{60}}{2} = \frac{8 \pm \sqrt{4}\sqrt{15}}{2}$

$x = \frac{8 \pm 2\sqrt{15}}{2} = \frac{8}{2} \pm \frac{2\sqrt{15}}{2} = \boxed{4 \pm \sqrt{15}}$

12. Use the quadratic formula to find the zeros.

$f(x) = -x^2 - 10x + 9$

$x = \frac{-(-10) \pm \sqrt{(-10)^2 - 4(-1)(9)}}{2(-1)}$

$x = \frac{10 \pm \sqrt{100 + 36}}{-2} = \frac{10 \pm \sqrt{136}}{-2} = \frac{10 \pm \sqrt{4}\sqrt{34}}{-2}$

$x = \frac{10 \pm 2\sqrt{34}}{-2} = \frac{-10}{-2} \pm \frac{-2\sqrt{34}}{2}$

$x = 5 \pm \sqrt{34} = \boxed{5 \pm \sqrt{34}}$

13. Graph. $f(x) = x^2 - 2x - 3$

Vertex: $x = \frac{-(-2)}{2(1)} = \frac{2}{2} = 1$

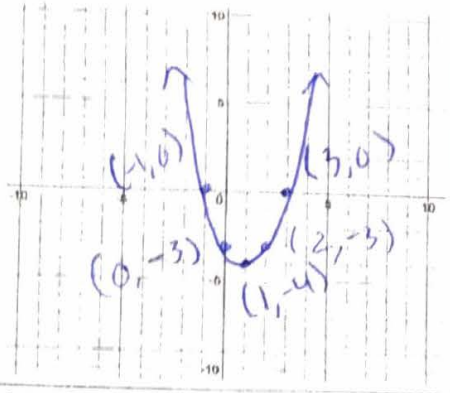
$y = (1)^2 - 2(1) - 3 = 1 - 2 - 3 = -4$

$V(1, -4)$

y-int $(0, -3)$

over 1 up 1, over 1 up 4

over 1 down 1, over 1 down 1



14. Graph. $f(x) = -x^2 - 6x - 8$

Vertex: $x = \frac{-(-6)}{2(-1)} = \frac{6}{-1} = -3$

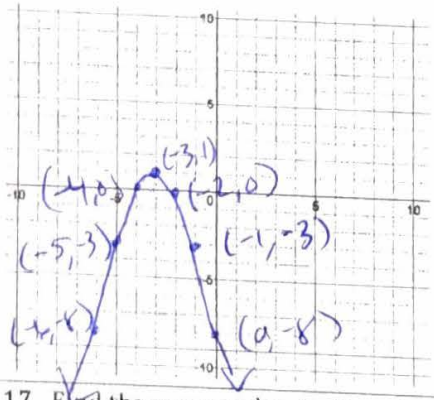
$y = -(-3)^2 - 6(-3) - 8$

$y = -9 + 18 - 8 = 9 - 8 = 1$

$(-3, 1)$

y-int $(0, -8)$

over 1 down 1, over 2 down 4



15. Graph. $f(x) = -x^2 - 8x - 7$

Vertex: $x = \frac{-(-8)}{2(-1)} = \frac{8}{-2} = -4$

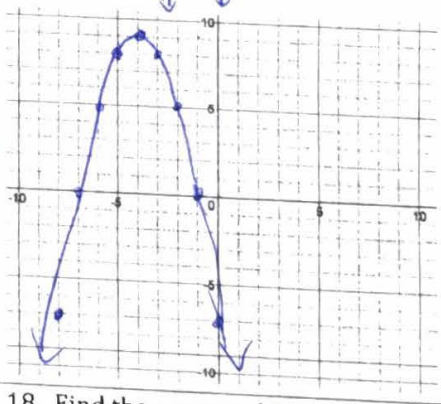
$y = -(-4)^2 - 8(-4) - 7$

$y = -16 + 32 - 7 = 9$

Vert. $(-4, 9)$

y-int $(0, -7)$

over 1 down 1, over 2 down 4, over 3 down 9



16. Find the zeros and write them in interval notation.

$f(x) < x^2 - 2x - 3$

(See graph above)

$(-1, 0)$ | $(3, 0)$ $<$ is below

So, outside & dotted:

$(-\infty, -1) \cup (3, \infty)$

17. Find the zeros and write them in interval notation.

$f(x) \leq -x^2 - 6x - 8$

(See graph above)

$\leq$ is below inside & solid

$[-4, -2]$

18. Find the zeros and write them in interval notation.

$f(x) \geq -x^2 - 8x - 7$

(See graph above) $\geq$ above solid

$(-\infty, -7] \cup [-1, \infty)$

Name: _____ Per: ____

19. Solve the system of equations algebraically over the set of real numbers.

$$\begin{cases} y = 2x - 5 \\ y = 4x^2 + 10x - 17 \end{cases}$$

$-y = -2x + 5$
$y = 4x^2 + 10x - 17$
A → $0 = 4x^2 + 8x - 12$
$x = \frac{-8 \pm \sqrt{(8)^2 - 4(4)(-12)}}{2(4)}$
$x = \frac{-8 \pm \sqrt{64+192}}{8} = \frac{-8 \pm \sqrt{256}}{8}$
$x = \frac{-8 \pm 16}{8} \rightarrow \frac{-8+16}{8} = \frac{8}{8} = 1$; $\frac{-8-16}{8} = \frac{-24}{8} = -3$ ← B
plug into $y = 2x - 5$
$y = 2(1) - 5 = 2 - 5 = -3$ → (1, −3)
$y = 2(-3) - 5$
$y = -6 - 5 = -11$ → (−3, −11)

20. Solve the system of equations algebraically over the set of real numbers.

$$\begin{cases} y = -3x + 7 \\ y = 2x^2 + 9x + 7 \end{cases}$$

$-y = +3x - 7$
$y = 2x^2 + 9x + 7$
$0 = 2x^2 + 12x + 0$ ← A
$x = \frac{-12 \pm \sqrt{(12)^2 - 4(2)(0)}}{2(2)}$
$x = \frac{-12 \pm \sqrt{144 - 0}}{4}$
$x = \frac{-12 \pm \sqrt{144}}{4} = \frac{-12 \pm 12}{4}$
$\frac{-12+12}{4} = \frac{0}{4}$ ← B, 2, 0
$\frac{-12-12}{4} = \frac{-24}{4} = -6$
Plug in: $y = -3x + 7$
$y = -3(0) + 7 = 7$ → (0, 7)
$y = -3(-6) + 7 = 18 + 7 = 25$ → (−6, 25)

21. Solve the system of equations algebraically over the set of real numbers.

$$\begin{cases} y = 5x + 1 \\ y = 3x^2 + 5x - 11 \end{cases}$$

$-y = -5x - 1$
$y = 3x^2 + 5x - 11$
$0 = 3x^2 + 0x - 12$ ← A
$x = \frac{-0 \pm \sqrt{(0)^2 - 4(3)(-12)}}{2(3)}$
$x = \frac{0 \pm \sqrt{0 + 144}}{6}$
$x = \frac{0 \pm \sqrt{144}}{6} = \frac{0 \pm 12}{6} = \frac{\pm 12}{6}$
$x = \frac{12}{6}$ or $\frac{-12}{6} = 2$ or -2 ← B
$y = 5(-2) + 1 = -10 + 1 = -9$ → (−2, −9)
$y = 5(2) + 1 = 10 + 1 = 11$ → (2, 11)

22. Simplify each expression by using i.

a. $i^{47} = -i$ ($i^{44} \cdot i^{45} \cdot i^{46} \cdot i^{47}$; $i^{44} = 1$)

b. $\sqrt{-64} = i\sqrt{64} = 8i$

c. $5 + \sqrt{-147} = 5 + i\sqrt{147} = 5 + i\sqrt{3}\sqrt{49} = 5 + 7i\sqrt{3}$

d. $(2 + 3i)(4 - 9i)$

	2	$+3i$
4	8	$+12i$
$-9i$	$-18i$	$-27i^2$

$8 + 12i - 18i - 27i^2$
$8 - 6i - 27(-1)$
$8 - 6i + 27 = 35 - 6i$

23. Simplify each expression by using i.

a. $i^{77} = i$ ($i^{76} \cdot i^{77}$; $i^{76} = 1$)

b. $\sqrt{-169} = i\sqrt{169} = 13i$

c. $2 + \sqrt{-128} = 2 + i\sqrt{128}$
$2 + i\sqrt{64}\sqrt{2} = 2 + 8i\sqrt{2}$

d. $(5 - i)(2 + 4i)$

	5	$-i$
2	10	$-2i$
$+4i$	$20i$	$-4i^2$

$10 - 2i + 20i - 4i^2$
$= 10 + 8i - 4(-1)$
$= 10 + 8i + 4 = 14 + 8i$

24. Simplify each expression by using i.

a. $i^{66} = -1$ ($i^{64} \cdot i^{65} \cdot i^{66}$; $i^{64} = 1$)

b. $\sqrt{-625} = i\sqrt{625} = 25i$

c. $7 - \sqrt{-243}$
$= 7 - i\sqrt{243} = 7 - i\sqrt{81}\sqrt{3} = 7 - 9i\sqrt{3}$

d. $(11 - 2i)(5 + 3i)$

	11	$-2i$
5	55	$-10i$
$+3i$	$33i$	$-6i^2$

$55 - 10i + 33i - 6i^2$
$55 + 23i - 6(-1)$
$55 + 23i + 6$
$61 + 23i$

25. List *all* words from the box that describe the number, $5 + 3i$.

Natural Number →	Whole Number →	Integer
Rational Number	Irrational Number →	Real Number
Imaginary Number	Complex Number	

Complex

26. List *all* words from the box that describe the number, 19.5.

Natural Number	Whole Number	Integer
Rational Number	Irrational Number	Real Number
Imaginary Number	Complex Number	

27. List *all* words from the box that describe the number, $\sqrt{7}$.

Natural Number	Whole Number	Integer
Rational Number	Irrational Number	Real Number
Imaginary Number	Complex Number	

Name: _____ Per: ____

28. A ball on an unknown planet is tossed upward from a height of 84 feet with an initial vertical velocity of 7 feet per second. Its height can be modeled by the quadratic function $h(t) = -7x^2 + 7x + 84$, where $h(t)$ is the height, in feet, of the ball, and t is the time the ball has been in the air, in seconds.

a. Determine the values of a, b, and c.

$a=-7 \quad b=7 \quad c=84$

b. How long will it take for the ball to reach the ground after it has been tossed? Round to the nearest hundredth.

$\frac{-7 \pm \sqrt{(7)^2-4(-7)(84)}}{2(-7)}$

$\frac{-7 \pm \sqrt{49+2352}}{-14}$

$\frac{-7 \pm \sqrt{2401}}{-14}$

$\frac{-7 \pm 49}{-14} = \frac{-7}{-14} \pm \frac{49}{-14} = \frac{-7-49}{-14}$ OR $\frac{-56}{-14}=4$

$\frac{-7+49}{-14} = \frac{42}{-14} = -3$ ✗

4 seconds

c. Find the maximum height the ball will reach.

That's the y of the vertex.

$x = \frac{-b}{2a} = \frac{-7}{2(-7)} = \frac{-7}{-14}$

$x = \frac{1}{2}$

$y = -7\left(\frac{1}{2}\right)^2 + 7\left(\frac{1}{2}\right) + 84$

$y = -7\left(\frac{1}{4}\right) + \frac{7}{2}\cdot\frac{2}{2} + 84\cdot\frac{4}{4}$

$y = -\frac{7}{4} + \frac{14}{4} + \frac{336}{4} = \frac{7}{4} + \frac{336}{4}$

$y = \frac{343}{4} =$ 85.75 feet

29. A ball on an unknown planet is tossed upward from a height of 60 feet with an initial vertical velocity of 48 feet per second. Its height can be modeled by the quadratic function $h(t) = -12x^2 + 48x + 60$, where $h(t)$ is the height, in feet, of the ball, and t is the time the ball has been in the air, in seconds.

a. Determine the values of a, b, and c.

$a=-12 \quad b=48 \quad c=60$

b. How long will it take for the ball to reach the ground after it has been tossed? Round to the nearest hundredth.

$x = \frac{-48 \pm \sqrt{(48)^2-4(-12)(60)}}{2(-12)}$

$x = \frac{-48 \pm \sqrt{2304+2880}}{-24}$

$x = \frac{-48 \pm \sqrt{5184}}{-24}$

$x = \frac{-48 \pm 72}{-24} = \frac{-48}{-24} \pm \frac{72}{-24}$

$x = +2 \pm 3$

$2+3=5 \quad 2-3=-1$

5 seconds

c. Find the maximum height the ball will reach.

vertex:

$x = \frac{-48}{2(-12)} = \frac{-48}{-24} = 2$

$-12(2)^2 + 48(2) + 60$

$-12(4) + 96 + 60$

$-48 + 96 + 60$

$48 + 60$

108 feet

30. A ball on an unknown planet is tossed upward from a height of 88 feet with an initial vertical velocity of 22 feet per second. Its height can be modeled by the quadratic function $h(t) = -11x^2 + 22x + 88$, where $h(t)$ is the height, in feet, of the ball, and t is the time the ball has been in the air, in seconds.

a. Determine the values of a, b, and c.

$a=-11 \quad b=22 \quad c=88$

b. How long will it take for the ball to reach the ground after it has been tossed? Round to the nearest hundredth.

$x = \frac{-22 \pm \sqrt{(22)^2-4(-11)(88)}}{2(-11)}$

$x = \frac{-22 \pm \sqrt{484+3872}}{-22}$

$x = \frac{-22 \pm \sqrt{4356}}{-22}$

$x = \frac{-22 \pm 66}{-22} = 1 \pm 3$

$1+3=4$ or $1-3=-2$

4 seconds

c. Find the maximum height the ball will reach.

vertex:

$x = \frac{-22}{2(-11)} = \frac{-22}{-22} = 1$

$y = -11(1)^2 + 22(1) + 88$

$= -11(1) + 22 + 88$

$= -11 + 22 + 88$

$= 11 + 88$

99 feet

28. A.

$A = -7 \quad B = -7 \quad C = 84$

$h(t) = -7t^2 - 7t + 84$

b.

$x = \frac{-7 \pm \sqrt{(7)^2 - 4(-7)(84)}}{2(-7)}$

AAA H!

$h(t) = -7t^2 + 7t + 84$

$\frac{0}{0}7 \quad \frac{0}{0}7 \quad \frac{0}{0}7$

$h(t) = 7(-t^2 + 1t + 12)$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-(1) \pm \sqrt{(1)^2 - 4(-1)(12)}}{2(-1)}$

$x = \frac{-1 \pm \sqrt{1+48}}{-2}$

$x = \frac{-1 \pm \sqrt{49}}{-2} = \frac{-1 \pm 7}{-2}$

$\frac{-1+7}{-2} = \frac{6}{-2} = -3$

$y \; \frac{-1-7}{-2}$

$\frac{-8}{-2} = 4$

4 secs

C. Maximum height (Vertex)

ORIGINAL → $x = \frac{-b}{2a} = \frac{-(7)}{2(-7)}$

$x = \frac{-7}{-14} = \frac{1}{2} = 0.5$

$y = -7(0.5)^2 + 7(0.5) + 84$

$-7(0.25) + 3.5 + 84$

$-1.75 + 3.50 + 84.00$

$y = 85.75$

Max height 85.75 feet